「学問に王道なし」

23 (1)

$$\begin{array}{r} 2x^2+3x+8 \\ 2x-3\overline{)\,4x^3\quad +7x+3} \\ 4x^3-6x^2 \\ \hline 6x^2+7x \\ 6x^2-9x \\ \hline 16x+3 \\ 16x-24 \\ \hline 27 \end{array}$$

商 $2x^2+3x+8$ 余り 27

(2) 求める整式をBとすると

$x^3-x^2+3x+1=B(x-2)+2x+7$

これを変形すると

$B(x-2)=x^3-x^2+3x+1-2x-7$

$=x^3-x^2+x-6$

よって

$B=(x^3-x^2+x-6)\div(x-2)$

$=x^2+x+3$

	1	−1	1	−6	2
		2	2	6	
	1	1	3	0	

24 (1)

$$\begin{array}{r} 5x\ +2a \\ x-3a\overline{)\,5x^2-13ax-6a^2} \\ 5x^2-15ax \\ \hline 2ax-6a^2 \\ 2ax-6a^2 \\ \hline 0 \end{array}$$

商 $5x+2a$ 余り 0

(2)

$$\begin{array}{r} 2x-a \\ 4x+3a\overline{)\,8x^2+2ax-2a^2} \\ 8x^2+6ax \\ \hline -4ax-2a^2 \\ -4ax-3a^2 \\ \hline a^2 \end{array}$$

商 $2x-a$ 余り a^2

(3)

$$\begin{array}{r} x^2-2yx-3y^2 \\ x+2y\overline{)\,x^3\quad -7y^2x-6y^3} \\ x^3+2yx^2 \\ \hline -2yx^2-7y^2x \\ -2yx^2-4y^2x \\ \hline -3y^2x-6y^3 \\ -3y^2x-6y^3 \\ \hline 0 \end{array}$$

商 $x^2-2xy-3y^2$ 余り 0

(4)

$$\begin{array}{r} 2x^2+yx+5y^2 \\ x-5y\overline{)\,2x^3-9yx^2\quad +10y^3} \\ 2x^3-10yx^2 \\ \hline yx^2 \\ yx^2-5y^2x \\ \hline 5y^2x+10y^3 \\ 5y^2x-25y^3 \\ \hline 35y^3 \end{array}$$

商 $2x^2+xy+5y^2$ 余り $35y^3$

「困難に打ち勝って輝く」

25 (1)
$$\begin{array}{r} x^2+yx-y^2 \\ 2x-y\ \overline{)\ 2x^3+yx^2-3y^2x-y^3} \\ 2x^3-yx^2 \\ \hline 2yx^2-3y^2x \\ 2yx^2-y^2x \\ \hline -2y^2x-y^3 \\ -2y^2x+y^3 \\ \hline 2y^3 \end{array}$$
商 x^2+xy-y^2
余り $2y^3$

(2)
$$\begin{array}{r} y^2+5xy+9x^2 \\ -y+2x\ \overline{)\ -y^3-3xy^2+x^2y+2x^3} \\ -y^3+2xy^2 \\ \hline -5xy^2+x^2y \\ -5xy^2+10x^2y \\ \hline -9x^2y+2x^3 \\ -9x^2y+18x^3 \\ \hline -16x^3 \end{array}$$
商 $y^2+5xy+9x^2$
余り $-16x^3$

26 (1) 求める整式Aは
$$A=(x^2-3x+5)(x^2+1)+3x-4$$
$$=x^4-3x^3+5x^2+x^2-3x+5+3x-4$$
$$=x^4-3x^3+6x^2+1$$

(2) 求める整式Bは次の等式を満たす。
$$x^4+1=B(x^2-x)+x+1$$
よって
$$B=(x^4-x)\div(x^2-x)=\frac{x(x^3-1)}{x(x-1)}=\frac{x(x-1)(x^2+x+1)}{x(x-1)}$$
$$=x^2+x+1$$

27
$$\begin{array}{r} x^2-yx+y^2 \\ x^2+yx+y^2\ \overline{)\ x^4+y^2x^2+y^4} \\ x^4+yx^3+y^2x^2 \\ \hline -yx^3 \\ -yx^3-y^2x^2-y^3x \\ \hline y^2x^2+y^3x+y^4 \\ y^2x^2+y^3x+y^4 \\ \hline 0 \end{array}$$
商 x^2-xy+y^2
余り 0

31 (1)
$$\frac{(-2ab)^2}{xy}\times\frac{x^4y^2}{-ab^2}=\frac{4a^2b^2}{xy}\times\frac{x^4y^2}{-ab^2}=-4abx^3y$$

(2)
$$\frac{x^2-y^2}{x^2-2xy+y^2}\div\frac{x^2+xy}{x-y}=\frac{(x+y)(x-y)}{(x-y)^2}\times\frac{(x-y)}{x(x+y)}=\frac{1}{x}$$

(3)
$$\frac{x}{x^2-8x+15}+\frac{x}{x^2-12x+35}=\frac{x}{(x-3)(x-5)}+\frac{x}{(x-5)(x-7)}$$
$$=\frac{x(x-7)+x(x-3)}{(x-3)(x-5)(x-7)}$$
$$=\frac{x(2x-10)}{(x-3)(x-5)(x-7)}$$
$$=\frac{2x(x-5)}{(x-3)(x-5)(x-7)}$$
$$=\frac{2x}{(x-3)(x-7)}$$

「千里の道も一歩から」

31 (4) $\dfrac{2x-1}{x^2-x-20}-\dfrac{2x+1}{x^2+x-30}$

$=\dfrac{2x-1}{(x+4)(x-5)}-\dfrac{2x+1}{(x-5)(x+6)}$

$=\dfrac{(2x-1)(x+6)-(2x+1)(x+4)}{(x+4)(x-5)(x+6)}$

$=\dfrac{2x^2+11x-6-2x^2-9x-4}{(x+4)(x-5)(x+6)}$

$=\dfrac{2(x-5)}{(x+4)(x-5)(x+6)}$

$=\dfrac{2}{(x+4)(x+6)}$

32 (1) $\dfrac{x-\frac{4}{x}}{1-\frac{2}{x}}=\dfrac{(x-\frac{4}{x})\times x}{(1-\frac{2}{x})\times x}=\dfrac{x^2-4}{x-2}=\dfrac{(x+2)(x-2)}{x-2}=x+2$

別

与式 $=(x-\frac{4}{x})\div(1-\frac{2}{x})=\dfrac{x^2-4}{x}\div\dfrac{x-2}{x}$

$=\dfrac{(x+2)(x-2)}{x}\times\dfrac{x}{x-2}=x+2$

(2) $\dfrac{\frac{1}{1-x}+\frac{1}{1+x}}{\frac{1}{1-x}-\frac{1}{1+x}}=\dfrac{(\frac{1}{1-x}+\frac{1}{1+x})\times(1-x)(1+x)}{(\frac{1}{1-x}-\frac{1}{1+x})\times(1-x)(1+x)}$

$=\dfrac{1+x+(1-x)}{1+x-(1-x)}$

$=\dfrac{2}{2x}=\dfrac{1}{x}$

別

分子÷分母

(3) $1-\dfrac{1}{1-\frac{1}{1-x}}=1-\dfrac{1\times(1-x)}{(1-\frac{1}{1-x})\times(1-x)}$

$=1-\dfrac{1-x}{1-x-1}$

$=1-\dfrac{1-x}{-x}=1+\dfrac{1-x}{x}$

$=\dfrac{x}{x}+\dfrac{1-x}{x}=\dfrac{1}{x}$

33 (1) $\dfrac{1}{x+y}-\dfrac{1}{x-y}+\dfrac{2x}{x^2-y^2}=\dfrac{1}{x+y}-\dfrac{1}{x-y}+\dfrac{2x}{(x+y)(x-y)}$

$=\dfrac{x-y-(x+y)+2x}{(x+y)(x-y)}$

$=\dfrac{2(x-y)}{(x+y)(x-y)}=\dfrac{2}{x+y}$

(2) $\dfrac{1}{x-1}+\dfrac{1}{x+1}-\dfrac{2x}{x^2+1}-\dfrac{4x}{x^4+1}$

$=\dfrac{x+1+x-1}{(x-1)(x+1)}-\dfrac{2x}{x^2+1}-\dfrac{4x}{x^4+1}$

$=\dfrac{2x}{x^2-1}-\dfrac{2x}{x^2+1}-\dfrac{4x}{x^4+1}$

$=\dfrac{2x(x^2+1)-2x(x^2-1)}{(x^2-1)(x^2+1)}-\dfrac{4x}{x^4+1}$

$=\dfrac{4x}{x^4-1}-\dfrac{4x}{x^4+1}$

$=\dfrac{4x(x^4+1)-4x(x^4-1)}{(x^4-1)(x^4+1)}$

$=\dfrac{8x}{x^8-1}$

「やればできる。君ならできる。」

33 (3) $\dfrac{c+a}{(a-b)(b-c)}+\dfrac{a+b}{(b-c)(c-a)}+\dfrac{b+c}{(c-a)(a-b)}$

$=\dfrac{(c+a)(c-a)+(a+b)(a-b)+(b+c)(b-c)}{(a-b)(b-c)(c-a)}$

$=\dfrac{c^2-a^2+a^2-b^2+b^2-c^2}{(a-b)(b-c)(c-a)}=0$

34 (1) $\dfrac{x+2}{x}-\dfrac{x+3}{x+1}-\dfrac{x-5}{x-3}+\dfrac{x-6}{x-4}$

$=\dfrac{(x+2)(x+1)-x(x+3)}{x(x+1)}-\dfrac{(x-5)(x-4)-(x-3)(x-6)}{(x-3)(x-4)}$

$=\dfrac{x^2+3x+2-(x^2+3x)}{x(x+1)}-\dfrac{x^2-9x+20-(x^2-9x+18)}{(x-3)(x-4)}$

$=\dfrac{2}{x(x+1)}-\dfrac{2}{(x-3)(x-4)}$

$=\dfrac{2(x-3)(x-4)-2x(x+1)}{x(x+1)(x-3)(x-4)}$

$=\dfrac{2(x^2-7x+12)-2(x^2+x)}{x(x+1)(x-3)(x-4)}$

$=\dfrac{-16x+24}{x(x+1)(x-3)(x-4)}$

(別) 与式$=\dfrac{x+2}{x}-\dfrac{(x+1)+2}{x+1}-\dfrac{(x-3)-2}{x+3}+\dfrac{(x-4)-2}{x-4}$

$=1+\dfrac{2}{x}-\left(1+\dfrac{2}{x+1}\right)-\left(1-\dfrac{2}{x+3}\right)+\left(1-\dfrac{2}{x-4}\right)$

$=\dfrac{2}{x}-\dfrac{2}{x+1}+\dfrac{2}{x+3}-\dfrac{2}{x-4}$

としておいてから計算してもよい。

(2) $\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+2)(x+3)}+\dfrac{1}{(x+3)(x+4)}$

$=\dfrac{(x+3)(x+4)+(x+1)(x+4)+(x+1)(x+2)}{(x+1)(x+2)(x+3)(x+4)}$

$=\dfrac{x^2+7x+12+x^2+5x+4+x^2+3x+2}{(x+1)(x+2)(x+3)(x+4)}$

$=\dfrac{3(x^2+5x+6)}{(x+1)(x+2)(x+3)(x+4)}=\dfrac{3(x+2)(x+3)}{(x+1)(x+2)(x+3)(x+4)}$

$=\dfrac{3}{(x+1)(x+4)}$

(別) $\dfrac{1}{(x+1)(x+2)}=\dfrac{1}{x+1}-\dfrac{1}{x+2}$, $\dfrac{1}{(x+2)(x+3)}=\dfrac{1}{x+2}-\dfrac{1}{x+3}$,

$\dfrac{1}{(x+3)(x+4)}=\dfrac{1}{x+3}-\dfrac{1}{x+4}$ なので

(与式)$=\left(\dfrac{1}{x+1}-\cancel{\dfrac{1}{x+2}}\right)+\left(\cancel{\dfrac{1}{x+2}}-\cancel{\dfrac{1}{x+3}}\right)+\left(\cancel{\dfrac{1}{x+3}}-\dfrac{1}{x+4}\right)$

$=\dfrac{1}{x+1}-\dfrac{1}{x+4}=\dfrac{x+4-(x+1)}{(x+1)(x+4)}=\dfrac{3}{(x+1)(x+4)}$

35 $\dfrac{1}{x(x+2)}+\dfrac{1}{(x+2)(x+4)}+\dfrac{1}{(x+4)(x+6)}$

$=\dfrac{(x+4)(x+6)+x(x+6)+x(x+2)}{x(x+2)(x+4)(x+6)}=\dfrac{3(x^2+6x+8)}{x(x+2)(x+4)(x+6)}$

$=\dfrac{3(x+2)(x+4)}{x(x+2)(x+4)(x+6)}=\dfrac{3}{x(x+6)}$

(別) $\dfrac{1}{x(x+2)}=\dfrac{1}{2}\left(\dfrac{1}{x}-\dfrac{1}{x+2}\right)$, $\dfrac{1}{(x+2)(x+4)}=\dfrac{1}{2}\left(\dfrac{1}{x+2}-\dfrac{1}{x+4}\right)$,

$\dfrac{1}{(x+4)(x+6)}=\dfrac{1}{2}\left(\dfrac{1}{x+4}-\dfrac{1}{x+6}\right)$ を用いても解ける。